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BREAKUP OF AN ANOMOLOUSLY VISCOUS LIQUID FILM IN A CENTRIFUGAL FORCE FIELD

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UDC 532.135

An equation is obtained for the breakup radius with consideration of tipping moments and Laplacian pressure forces acting on the liquid ridge at the critical point.

As is well known, in centrifugal atomizers breakup of the liquid film and droplet formation may occur beyond the edges of the cup, at its boundary, or on its surface. Experiments have shown that in the last case the droplet dispersion becomes more homogeneous. Study of liquid film breakup is also necessary to determine the minimum liquid flow density in film-type centrifugal devices [1].

The goal of the present study is to determine the critical breakup parameters (liquid film radius of depth) as functions of the technological parameters.

We will consider the breakup of a laminar isothermal film of an anomolously viscous liquid which obeys a power-type law on the surface of a curvilinear cup. Experiments have shown that a liquid ridge is formed at the boundary between the dry and wetted surface areas. We will describe the forces acting on the liquid ridge at the critical point G using the notation of [2] (Fig. 1). We assume that the ridge has a cylindrical surface with constant radius of curvature R_r.

Considering the phenomenon of wetting angle hysteresis (i.e., the possibility of short-term rotation of the liquid film surface about the critical point G), we write the equation for the equilibrium state of the ridge

$$M_\sigma + M_v + M_\omega = 0, \tag{1}$$

where

$$M_\sigma = \sigma\delta_c; \tag{2}$$

$$M_v = -\int_0^{\delta_c} \frac{\rho v_l^2}{2}\, \delta d\delta; \tag{3}$$

$$M_\omega = -\int_0^{h_0} \rho a\omega^2 r \sin\alpha\, h dh. \tag{4}$$

Assuming that the velocity profile is defined [3, 4] as

$$v_l = \beta\left(\frac{\omega r}{4}\right)\left(\frac{2n+1}{n+1}\right)\left[1-\left(1-\frac{\delta}{\delta_c}\right)^{\frac{n+1}{n}}\right], \tag{5}$$

we integrate Eq. (3), obtaining

$$M_v = -T_1\delta_c^2\left(\frac{\omega r}{4}\right)^2\beta^2, \tag{6}$$

where

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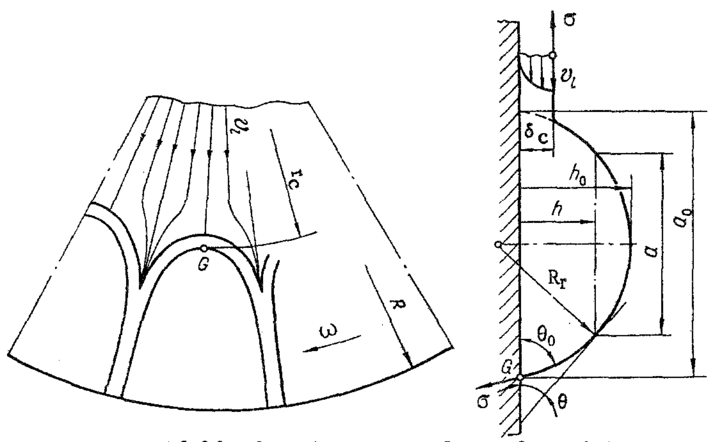

Fig. 1. Liquid film breakup on surface of a rotating cup.

$$T_1 = \frac{(21n^3 + 16n^2 + 5n + 2)(2n + 1)}{4(3n + 1)(3n + 2)(n + 1)^2}.$$

Considering the geometry of the ridge, we have:

$$h_0 = R_r(1 - \cos\theta_0);\ h = h_0 - R_r(1 - \cos\theta);$$
$$a = 2R_r \sin\theta;\ dh = -R_r \sin\theta\, d\theta. \tag{7}$$

Since at $h = h_0$ $\theta = 0$ and at $h = 0$ $\theta = \theta_0$, by using Eq. (7) we obtain from Eq. (4)

$$M_\omega = -T_2 R_r^3 \rho\omega^2 r \sin\alpha, \tag{8}$$

where

$$T_2 = \left(\sin\theta_0 - \theta_0\cos\theta_0 - \frac{1}{3}\sin^3\theta_0\right).$$

To determine R_r we use the Laplace equation [5], according to which the relationship between excess pressure beneath the ridge and ridge curvature may be represented in the form

$$p_r = \sigma/R_r. \tag{9}$$

The excess pressure beneath the ridge may be written as

$$p_r = p_{vm} + p_{\omega m} + p_{pm}, \tag{10}$$

where

$$p_{vm} = \frac{1}{\delta_c}\int_0^{\delta_c} \frac{\rho v_l^2}{2}\, d\delta; \tag{11}$$

$$p_{\omega m} = \frac{1}{h_0}\int_0^{h_0} \frac{1}{a}\int_0^{a} \rho a\omega^2 r \sin\alpha\, da\, dh; \tag{12}$$

$$p_{pm} = \frac{h_0\rho\omega^2 r}{2}\cos\alpha. \tag{13}$$

Equation (13) is derived from the expression for pressure obtained in [4] by replacing δ_0 by h_0 with the consideration that $\beta = 0$, $\psi = 0$ in the stagnant zone within the ridge. Integration of Eqs. (11), (12) with use of Eqs. (5), (7) gives:

$$p_{vm} = \rho\frac{2n + 1}{3n + 2}\left(\frac{\omega r}{4}\right)^2 \beta^2; \tag{14}$$

$$p_{\omega m} = \frac{\theta_0 - \sin\theta_0\cos\theta_0}{2(1 - \cos\theta_0)}\rho\omega^2 r \sin\alpha\, R_r. \tag{15}$$

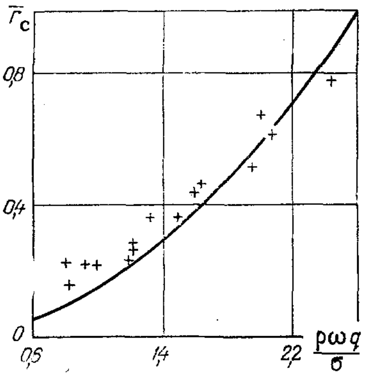

Fig. 2

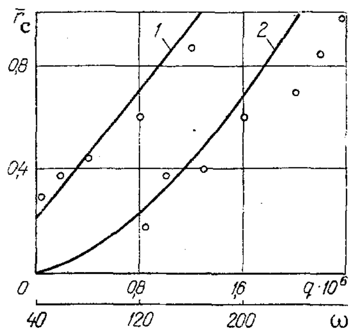

Fig. 3

Fig. 2. Dimensionless critical radius versus dimensionless complex $\rho\omega q/\sigma$ at $n = 1$, $K = 1.8 \cdot 10^{-1}$ N·sec/m²; $\rho = 1.23 \cdot 10^3$ kg/m³; $R = 12.5 \cdot 10^{-2}$ m; $\sigma = 48.7 \cdot 10^{-3}$ N/m; $\theta_0 = 1.223$ rad; $\alpha = \pi/2$ rad, $q = (0.55\text{–}1.7) \cdot 10^{-6}$ m³/sec; $\omega =$ 30–120 rad/sec.

Fig. 3. Dimensionless critical radius versus flow rate and angular velocity at $n = 0.77$, $K = 1.57 \cdot 10^{-1}$ N·secⁿ/m²; $\rho = 1.017 \cdot 10^3$ kg/m³; $R = 12.5 \cdot 10^{-2}$ m, $\theta_0 = 1.0248$ rad; $\sigma = 53.3 \cdot 10^{-3}$ N/m; $\alpha = \pi/2$ rad; 1) $\bar{r}_c = f(\omega)$ for $1 \cdot 10^{-6}$ m³/sec; 2) $\bar{r}_c = f(q)$ for $\omega = 60$ rad/sec.

Substituting Eqs. (10), (13)-(14) in Eq. (9), we obtain the Laplace equation in the form

$$p_{\upsilon m} + \zeta R_r + \eta R_r = \frac{\sigma}{R_r}, \tag{16}$$

where $\zeta = p_{\omega m}/R_r$; $\eta = p_{pm}/R_r$. It follows from Eq. (16) that

$$R_r^2 + R_r\left(\frac{p_{\upsilon m}}{\zeta + \eta}\right) - \frac{\sigma}{(\zeta + \eta)} = 0. \tag{17}$$

Considering that in the particular problem considered the range of variation of the technological parameters $R_r p_v/\sigma \sim q\rho v/\sigma R \ll 1$, in Eq. (17) we may neglect the second term in comparison to the third as being of second order smallness. Then

$$R_r = \sqrt{\frac{\sigma}{\zeta + \eta}} = T_3(\alpha, \theta_0)\left(\frac{\sigma}{\rho\omega^2 r}\right)^{0,5}, \tag{18}$$

where

$$T_3 = \left[\frac{2(1 - \cos\theta_0)}{\sin\alpha\,(\theta_0 - \sin\theta_0\cos\theta_0) + (1 - \cos\theta_0)^2\cos\alpha}\right]^{1/2}.$$

Substituting Eqs. (2), (6), (8), and (18) in Eq. (1), we obtain the moment equation in the form

$$\sigma\delta_c - T_1\rho\delta_c^2\left(\frac{\omega r}{4}\right)^2\beta^2 - T_4\left(\frac{\sigma^3}{\rho\omega^2 r}\right)^{0,5} = 0, \tag{19}$$

where

$$T_4 = T_2 T_3^3 \sin\alpha.$$

Since the second and third terms of Eq. (19) are in the ratio $J^2/We^{0.5} \ll 1$, the second term may be neglected. For the problem under consideration $J^2 \sim 1$ and $We^{0.5} \sim 10^2$. Here $J = \rho\omega q/\sigma$, while $We = \rho\omega^2 R^3/\sigma$ is the Weber criterion. Then from Eq. (19)

$$\delta_c = T_4(\sigma/\rho\omega^2 r)^{0,5}. \tag{20}$$

If we neglect the tangential velocity of the liquid ($\psi = 0$), then to an accuracy of 3%, following [3], the film thickness will be given by

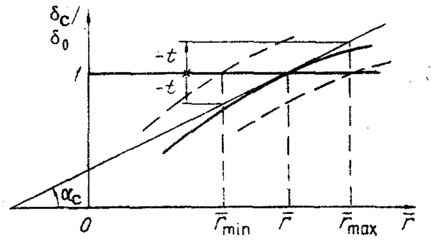

Fig. 4. Determination of zone of deviation of critical radius from calculated value.

$$\delta_0 = \left[\left(\frac{2n+1}{2\pi n} \right)^n \frac{Kq^n}{\rho\omega^2 r^{n+1} \sin\alpha} \right]^{\frac{1}{2n+1}}. \tag{21}$$

Simultaneous solution of Eqs. (20) and (21) makes it possible to determine the breakup radius of the liquid film

$$\bar{r}_c = \left[T(\theta_0, \alpha, n) \left(\frac{\rho\omega q}{\sigma} \right)^n \left(\frac{\omega^n KR}{\sigma} \right) \frac{1}{\mathrm{We}^{0.5}} \right]^2, \tag{22}$$

where

$$T(\theta_0, \alpha, n) = \left(\frac{2n+1}{2\pi n} \right)^n \frac{\sin\alpha}{T_4^{2n+1}}.$$

If necessary, the problem may also be solved without using the above simplifications, by numerical methods. Equation (22) can then serve as a first approximation.

To verify the theoretically obtained functions, experiments were performed on a plane disk with a 90% aqueous solution of glycerine (Fig. 2) (Newtonian liquid) and for a 2.5% aqueous solution of KMTs-600 (power-like non-Newtonian liquid, Fig. 3).

The parameters K and n were determined by the two-capillary method in a constant pressure viscosimeter; ρ, by a densimeter; σ, by the ring breakoff method; and θ_0, by the liquid ascent level when wetting the specimen of [6].

Since there were significant deviations in breakup radius in each experiment (Δ = 15-20%) mean arithmetic values were taken.

As is evident from the curves, the experimental values deviate from the theoretical ones by not more than 25-30%. It is evident from Fig. 3 that the divergence between experiment and theory for small $\bar{r}_c$ reaches 100%. This is apparently the result of the simplifications and assumptions used, both those as to flow hydrodynamics and those on liquid film breakup. At small radii these have a greater effect.

To estimate the deviation of the breakup radius from the calculated value, we write the solution of Eqs. (20), (21) in the following form:

$$\frac{\delta_c}{\delta_0} - \left(\frac{\bar{r}}{\bar{r}_c} \right)^{\frac{1}{4n+2}} = 0, \quad \frac{\delta_c}{\delta_0} - 1 = 0. \tag{23}$$

We then assume that random factors not considered by the theory lead to some oscillation of the ratio δ_c/δ_0 by by an amount ±t. This then corresponds to variations $\bar{r}_{max}$ and $\bar{r}_{min}$ (Fig. 4). If we denote the deviation range by

$$\Delta = (\bar{r}_{max} - \bar{r}_{min})/2\bar{r}_c,$$

and consider that for the slope of the tangent at the critical point we have

$$\mathrm{tg}\,\alpha_c = \frac{1}{2(2n+1)\bar{r}_c} = \frac{t}{\bar{r}_c \cdot \Delta},$$

we then obtain

$$\Delta = \pm 2t(2n+1). \tag{24}$$

It follows from Eq. (24) that if n = 1 and t = ±0.03, then Δ = ±0.18. Thus, small oscillations of the ratio δ_c/δ_0(3%) can lead to significant deviations of the breakup radius (±18%), which corresponds to the phenomenon observed.

NOTATION

K, n, rhenological constants; ρ, density; σ, surface tension; r, current cup radius; R, maximum cup radius; r_c, critical radius for film breakup; $\bar{r} = \bar{r} = r/R$, dimensionless current radius; $\bar{r}_c = r_c/R$, dimensionless critical radius; δ_0, δ_c, actual and critical film thicknesses; δ, current thickness; R_r, ridge radius; h_0, ridge height; h, current ridge height; θ_0, limiting wetting angle; θ, current angle of tangent to ridge surface; α, angle between axis of rotation and tangent to cup surface; ω, angular velocity of rotation; q, volume liquid flow rate; v_1 and v_φ, meridional and tangential velocities; $\beta = 4v_{lm}/\omega r$, $\psi = 4v_{\varphi m}/\omega r$, dimensionless velocities; M_σ M_ω, moments of surface and centrifugal forces; M_v, moment from velocity head; p_r, pressure within ridge; p_{vm}, pressure from velocity head; $p\omega_m$, p_{pm}, pressures from centrifugal force components tangent and normal to cup surface; Δ, deviation range of breakup radius from calculated value; $\bar{r}_{max}$, $\bar{r}_{min}$, limiting deviations of breakup radius; α_c, angle of tangent to curve $\delta_c/\delta_0 = f(\bar{r})$ at critical point; t, random oscillation of ratio δ_c/δ_0.

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LAMINAR FLOW OF A VISCOUS INCOMPRESSIBLE LIQUID OVER THE SURFACE OF SOLIDS OF REVOLUTION

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The laminar flow of a viscous incompressible liquid over the surface of stationary solids of revolution is examined for the case of circular inflow of the stream.

Dispersing agents are presently used in industry [1-6] whereby a stream (jet) of viscous liquid spreads in an axisymmetric thin layer over the surface of stationary solids of revolution of various shapes. Several works [7-13] have been devoted to the study of such laminar flows. However, the results here were obtained without regard for the parameters of the inflowing stream, which leads to the appearance of quantities in the theoretical data whose values can be found only by experiment—a serious deficiency of these researches.

There are studies [14-16] which have overcome this problem.

This article examines the axisymmetric, stable thin-layer flow of a viscous, incompressible, uniform liquid over the curvilinear surface of solids of revolution in the laminar mode as a result of the inflow of an infinite circular stream, with allowance for the working parameters of the latter.

We will examine flow of the liquid in the special system of coordinates l, η, and θ. The given coordinate system is orthogonal (Fig. 1).

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